

**THREE-BODY SCATTERING WITH  
TWO-CHARGED PARTICLES:  
APPLICATION TO DIRECT  
NUCLEAR REACTIONS**

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- For more than 30 Years the application of Exact Three-Body theory has been shadowed by the difficulty in including the Coulomb interaction between charged particles.
- In 2005 the situation changed due to the work on  $p - d$  elastic scattering and breakup by A. Deltuva, A. C. Fonseca, and P. U. Sauer
- Continuum Discretized Coupled Channel (CDCC) calculations have been the most important tool to cope with reactions where coupling to breakup is an important part of the reaction mechanism.

Momentum-Space Treatment of Coulomb Interaction in Three-Nucleon Reactions with Two Protons,

Phys. Rev. C71, 054005 (2005).

Benchmark Calculation for Proton-Deuteron Elastic Scattering Observables Including Coulomb,

Phys. Rev. C71, 064003 (2005).

Calculation of Proton-Deuteron Breakup Reactions Including the Coulomb Interaction Between the Two Protons,

Phys. Rev. Lett. 95, 092301 (2005).

Momentum-Space Description of Three-Nucleon Breakup Reactions Including the Coulomb Interaction,

Phys. Rev. C72, 054004 (2005).

# GOAL

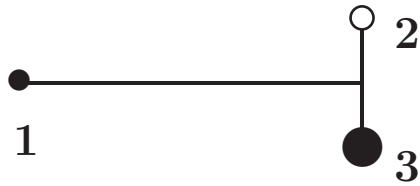
- Use exact Integral Equations of Alt, Grassberger and Sandhas (AGS);
- Realistic Forces between all pairs including optical potentials;
- Full treatment of the Coulomb interaction using Screened Coulomb + Renormalization (Taylor, Alt, Sandhas);
- Precise numerics to solve AGS equations with large number of partial waves;
- Tackle all possible reactions.

## *p* – *B* ELASTIC SCATTERING AND BREAKUP

$$B = A + n$$

*A* is heavy nucleus

## Three-body odd man out notation



$$\alpha = 1, 2 \text{ or } 3$$

$v_\alpha$  = hadronic pair interaction

$\omega_{\alpha R}$  = screened Coulomb pair interaction

If pair

$$\alpha = np/nA \quad \omega_{\alpha R} = 0$$

$$\alpha = pA \quad \omega_{\alpha R} \neq 0$$

We screen the Coulomb Potential

$$w_R(r) = w(r) e^{-(r/R)^n},$$

$$w(r) = Z_1 Z_2 \alpha_e / r \quad \alpha_e \approx 1/137.$$

$n = 1 \Rightarrow$  Yukawa Screening used by Alt et al.

$n = 6 \Rightarrow$  Our optimal choice for nuclear reactions  
( $n = 4$  in pd work)

The full t-matrix for  $v + w_R$  is defined as

$$t^{(R)}(z) = (v + w_R) + (v + w_R)g_0(z)t^{(R)}(z).$$

For the screened Coulomb potential we may  
define

$$t_R(z) = w_R + w_R g_0(z)t_R(z).$$

Let

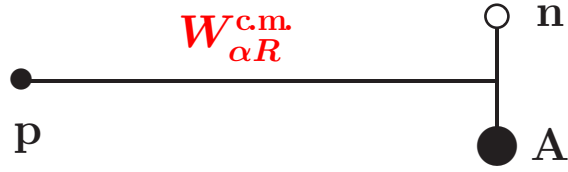
$$U_{\beta\alpha}^{(R)}(\mathbf{Z}) = \bar{\delta}_{\beta\alpha} G_0^{-1}(\mathbf{Z}) + \sum_{\sigma} \bar{\delta}_{\beta\sigma} T_{\sigma}^{(R)}(\mathbf{Z}) G_0(\mathbf{Z}) U_{\sigma\alpha}^{(R)}(\mathbf{Z}),$$

$$T_{\alpha}^{(R)}(\mathbf{Z}) = (v_{\alpha} + w_{\alpha R}) + (v_{\alpha} + w_{\alpha R}) G_0(\mathbf{Z}) T_{\alpha}^{(R)}(\mathbf{Z}),$$

be the AGS equation for particle-pair scattering, and

$$U_{0\alpha}^{(R)}(\mathbf{Z}) = G_0^{-1}(\mathbf{Z}) + \sum_{\sigma} T_{\sigma}^{(R)}(\mathbf{Z}) G_0(\mathbf{Z}) U_{\sigma\alpha}^{(R)}(\mathbf{Z}),$$

for breakup.



Let  $W_{\alpha R}^{c.m.}$  be the screened Coulomb potential between one proton and the c.m. of the remaining pair, and  $T_{\alpha R}^{c.m.}$  the corresponding t-matrix.

$$W_{\alpha R}^{c.m.} = 0 \text{ if } \alpha \text{ is a pA pair.}$$

One may write

$$U_{\beta\alpha}^{(R)} = \underset{\text{long range}}{\delta_{\beta\alpha} T_{\alpha R}^{c.m.}} + \underset{\text{short range}}{[U_{\beta\alpha}^{(R)} - \delta_{\beta\alpha} T_{\alpha R}^{c.m.}]},$$

to subtract the singular part of  $U_{\beta\alpha}^{(R)}$ . If the remainder can be identified with a short range operator we may proceed as before.

$\tilde{U}_{\beta\alpha}^{(R)}$  and  $\tilde{U}_{0\alpha}^{(R)}$  are proved to be short range operators.

Using the renormalization prescription to reach the unscreened limit ( $R \rightarrow \infty$ ):

$$\begin{aligned}
& \langle \phi_\beta(\vec{q}_f) \nu_{\beta_f} | U_{\beta\alpha} | \phi_\alpha(\vec{q}_i) \nu_{\alpha_i} \rangle \\
&= \delta_{\beta\alpha} \langle \phi_\beta(\vec{q}_f) \nu_{\beta_f} | \mathbf{T}_{\alpha C}^{\text{c.m.}} | \phi_\alpha(\vec{q}_i) \nu_{\alpha_i} \rangle \\
&+ \lim_{R \rightarrow \infty} \{ \mathcal{Z}_{\beta R}^{-\frac{1}{2}}(q_f) \langle \phi_\beta(\vec{q}_f) \nu_{\beta_f} | \\
&\times [U_{\beta\alpha}^{(R)}(E_\alpha(q_i) + i0) - \delta_{\beta\alpha} \mathbf{T}_{\alpha R}^{\text{c.m.}}(E_\alpha(q_i) + i0)] \\
&\times | \phi_\alpha(\vec{q}_i) \nu_{\alpha_i} \rangle \mathcal{Z}_{\alpha R}^{-\frac{1}{2}}(q_i) \},
\end{aligned}$$

and

$$\begin{aligned}
& \langle \phi_0(\vec{p}_f, \vec{q}_f) \nu_{0_f} | U_{0\alpha} | \phi_\alpha(\vec{q}_i) \nu_{\alpha_i} \rangle \\
&= \lim_{R \rightarrow \infty} \{ z_R^{-\frac{1}{2}}(p_f) \langle \phi_0(\vec{p}_f, \vec{q}_f) \nu_{0_f} | \\
&\times U_{0\alpha}^{(R)}(E_\alpha(q_i) + i0) | \phi_\alpha(\vec{q}_i) \nu_{\alpha_i} \rangle \mathcal{Z}_{\alpha R}^{-\frac{1}{2}}(q_i) \}
\end{aligned}$$

$$\mathcal{Z}_{\alpha R}(q_i) = e^{-2i\kappa_\alpha(q_i)[\ln(2q_i R) - C/n]},$$

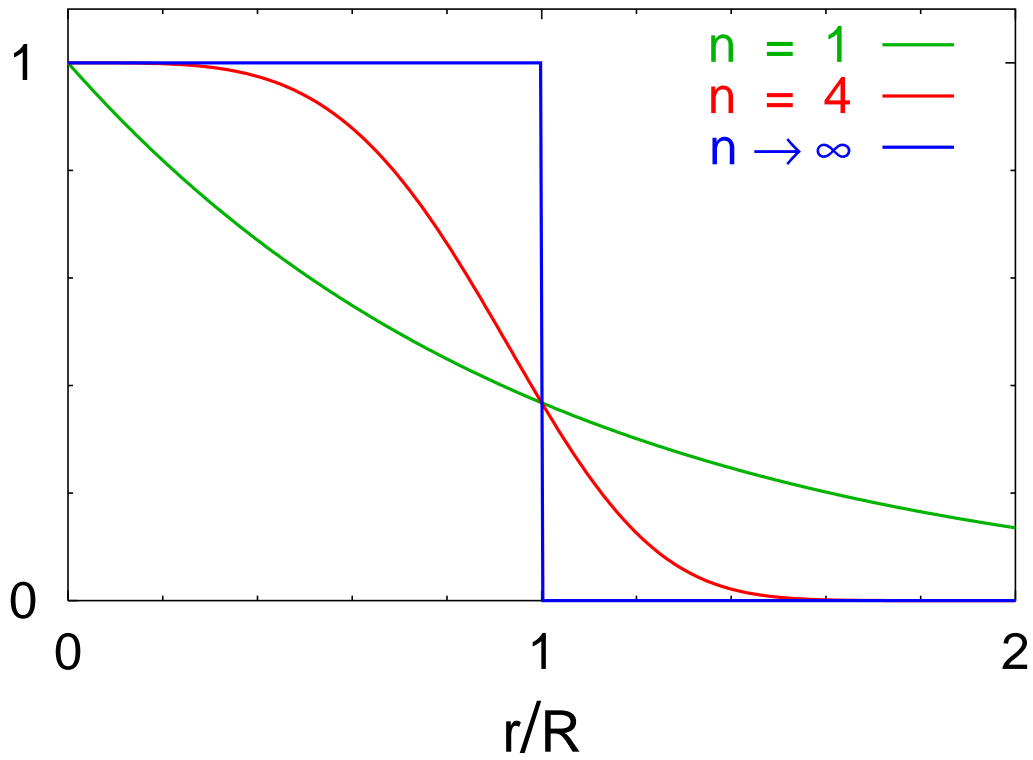
$$z_R(p_f) = e^{-2i\kappa(p_f)[\ln(2p_f R) - C/n]},$$

$\kappa_\alpha(q_i) = \alpha_e M_\alpha / q_i$     pB or Ad Coulomb  
parameters

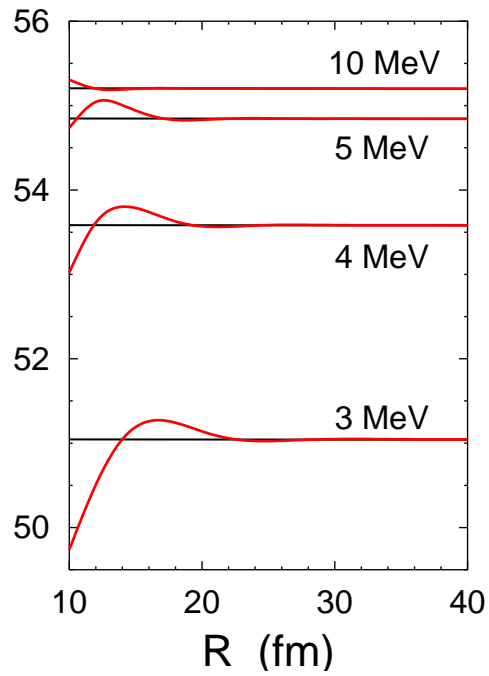
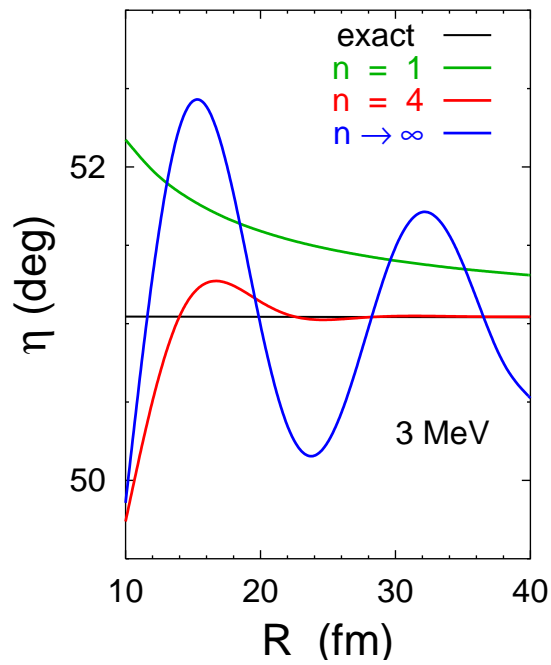
$\kappa(p_f) = \alpha_e \mu / p_f$     pA Coulomb parameters

# Screened Coulomb potential

$$\frac{w_R(r)}{w(r)} = e^{-\left(\frac{r}{R}\right)^n}$$



# $pp$ scattering: $^1S_0$ phase shift



**COMPARISON OF CDCC AND  
FADDEEV CALCULATIONS FOR  
 $^{11}\text{Be}-p$  SCATTERING**

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## THE QUESTION IS:

How reliable is CDCC compared to exact solutions for the same effective three-body problem.

## MODEL PROBLEM

$$p + {}^{10}\text{Be} + n \rightarrow \begin{cases} p - {}^{11}\text{Be} \text{ scattering} \\ p - {}^{11}\text{Be} \text{ breakup} \end{cases}$$

$p - {}^{10}\text{Be}$  An optical potential + Coulomb

$n - {}^{10}\text{Be}$  Real potential that describes

s – wave bound state

p – wave excited state

d – wave resonance.

$n - p$  Real potential that supports  
a bound state at the deuteron  
binding energy

# CDCC

## 1 - CDCC-BU

The wave function is expressed in terms of the continuum states of

$$^{10}\text{Be} + n.$$

This is direct breakup (BU) where



## 2 - CDCC-TR

The wave function is expressed in terms of the continuum states of

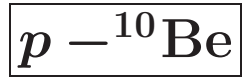
$$n - p.$$

This is the transfer of the neutron to the continuum of the deuteron (TR)



In Nucl. Phys. A767, 138 (2006) Moro and Filomena showed that breakup observables for both processes do not coincide.

## Interactions: Independent of Spin



$$V(r) = -V_o \left[ 1 + e^{\frac{r-R_o}{a_o}} \right]^{-1} - iW_v \left[ 1 + e^{\frac{r-R_v}{a_v}} \right]^{-1} \\ + \text{Coulomb}$$

$$V_o = 51.2 \text{ MeV}$$

$$W_v = 19.5 \text{ MeV}$$

$$R_o = r_o A^{\frac{1}{3}}$$

$$R_v = r_v A^{\frac{1}{3}}$$

$$r_o = 1.114 \text{ fm}$$

$$r_v = 1.114$$

$$a_o = 0.57 \text{ fm}$$

$$a_v = 0.50$$

$n - {}^{10}\text{Be}$

$$V(r) = -V_c \left[ 1 + e^{\frac{r-R_c}{a_c}} \right]^{-1}$$

$$R_c = r_c A^{\frac{1}{3}}$$

$L=0$       $V_c = 51.639 \text{ MeV}$

$r_c = 1.39 \text{ fm}$

$\epsilon_o^* = -0.503 \text{ MeV}$

$a_c = 0.52 \text{ fm}$

$(\epsilon_o = -30.28 \text{ MeV})$

removed

$L=1$       $V_c = 26.26 \text{ MeV}$

$r_c = 1.39 \text{ fm}$

$\epsilon_1 = -0.183 \text{ MeV}$

$a_c = 0.52 \text{ fm}$

$L \geq 2$       $V_c = 51.639 \text{ MeV}$

$r_c = 1.39 \text{ fm}$

$\epsilon_2 = 1.317 - i188/2$

$a = 0.52 \text{ fm}$

$n - p$

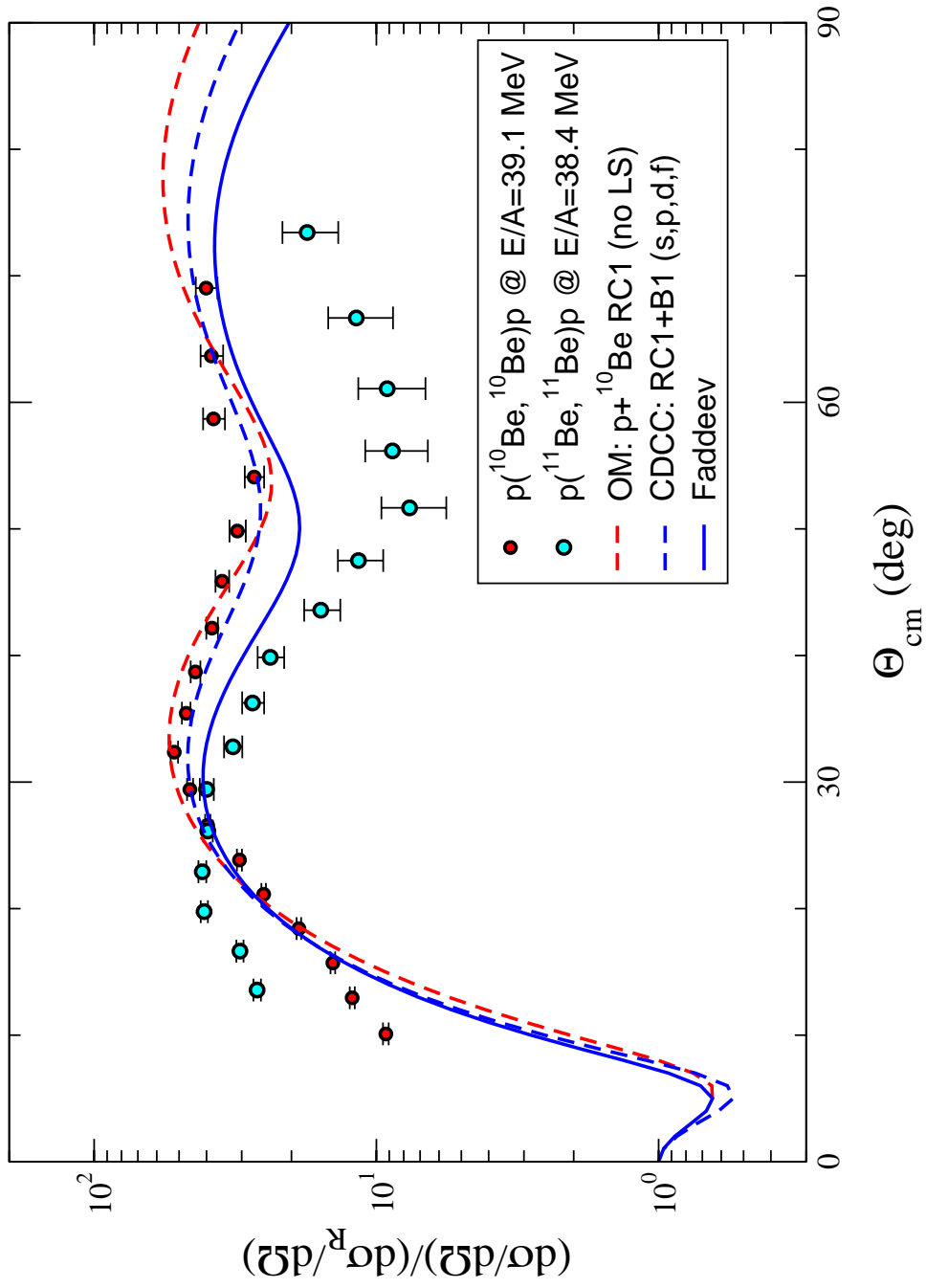
$$V(r) = -V_o e^{(-\frac{r}{r_o})^2}$$

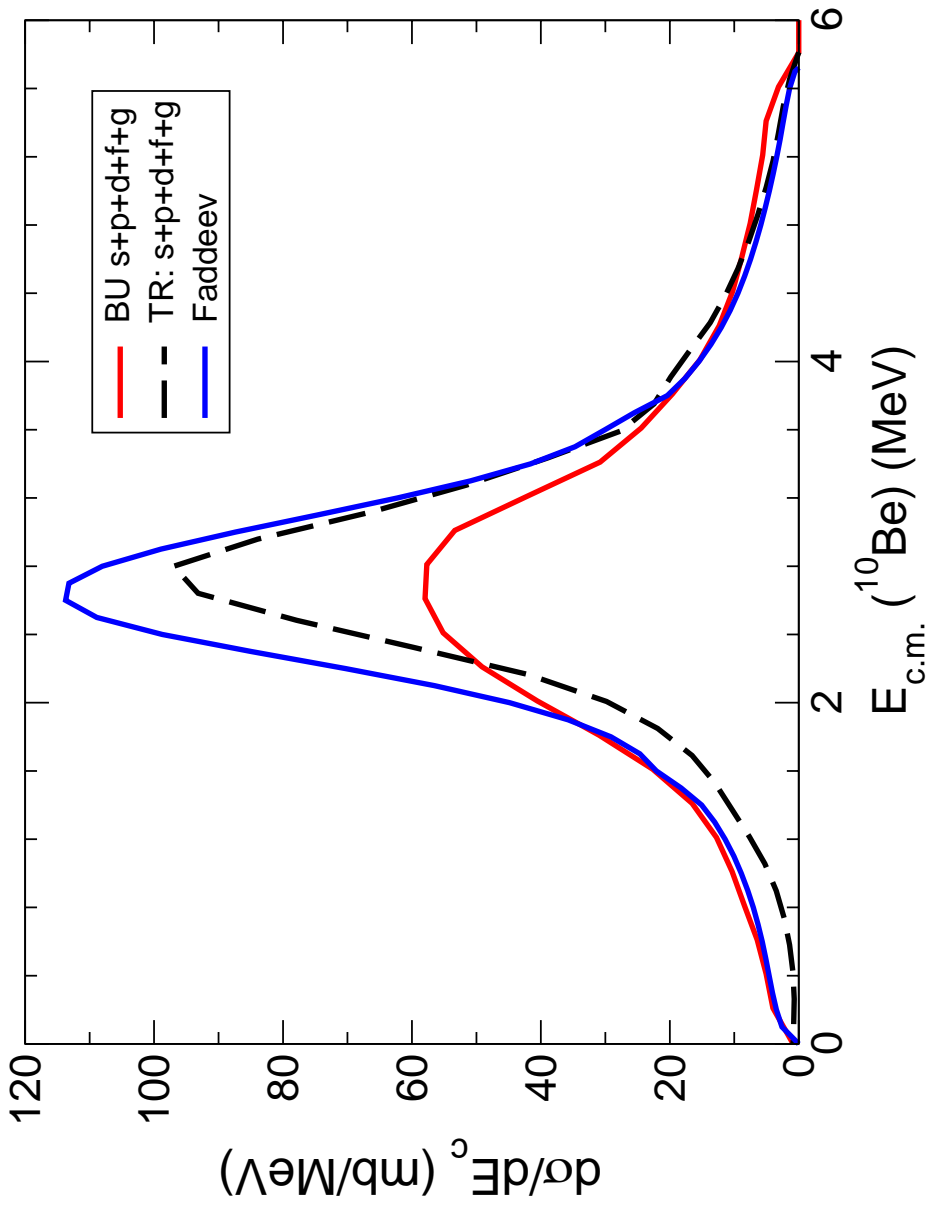
$$V_o = -72.15 \text{ MeV}$$

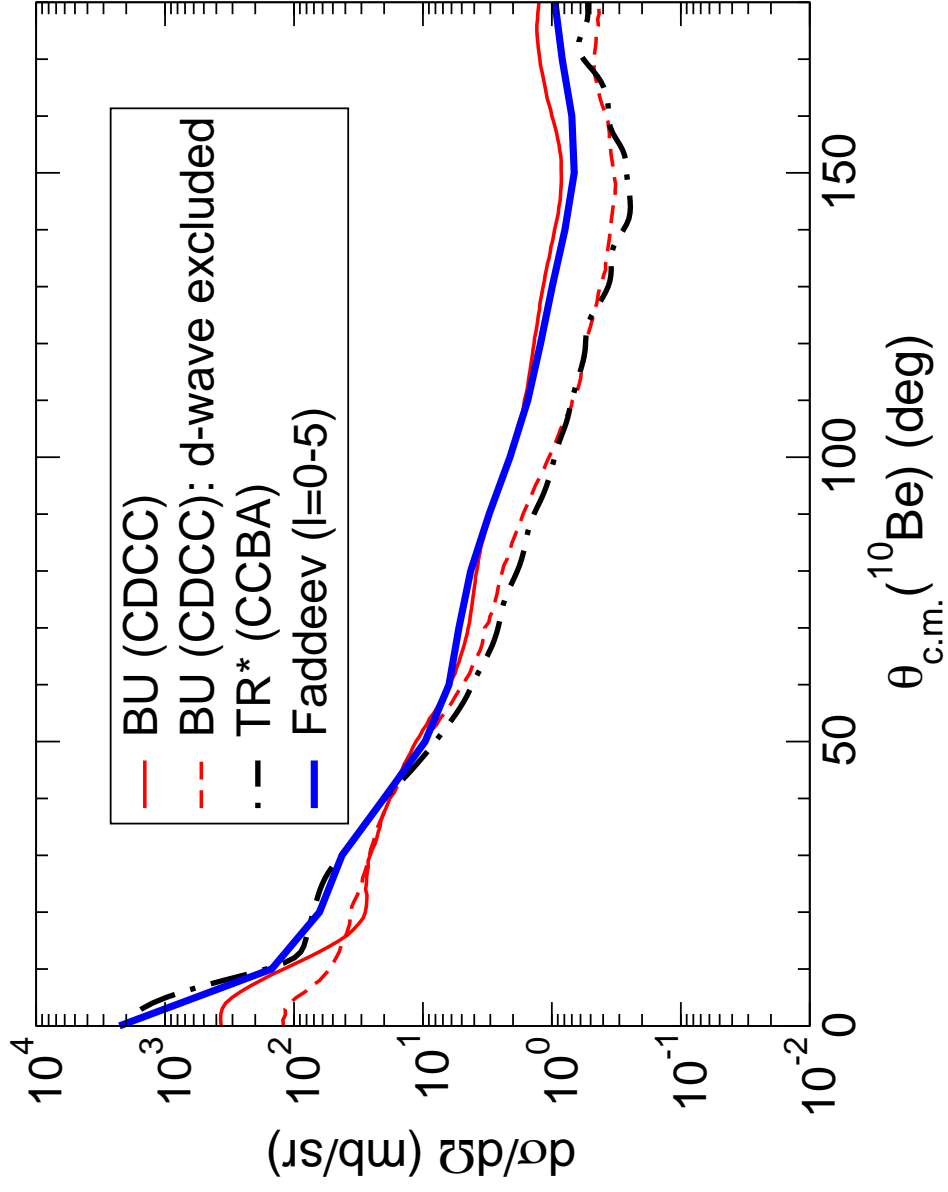
$$r_o = 1.484 \text{ fm}$$

$L=0$       bound state       $\epsilon_o = -2.224 \text{ MeV}$

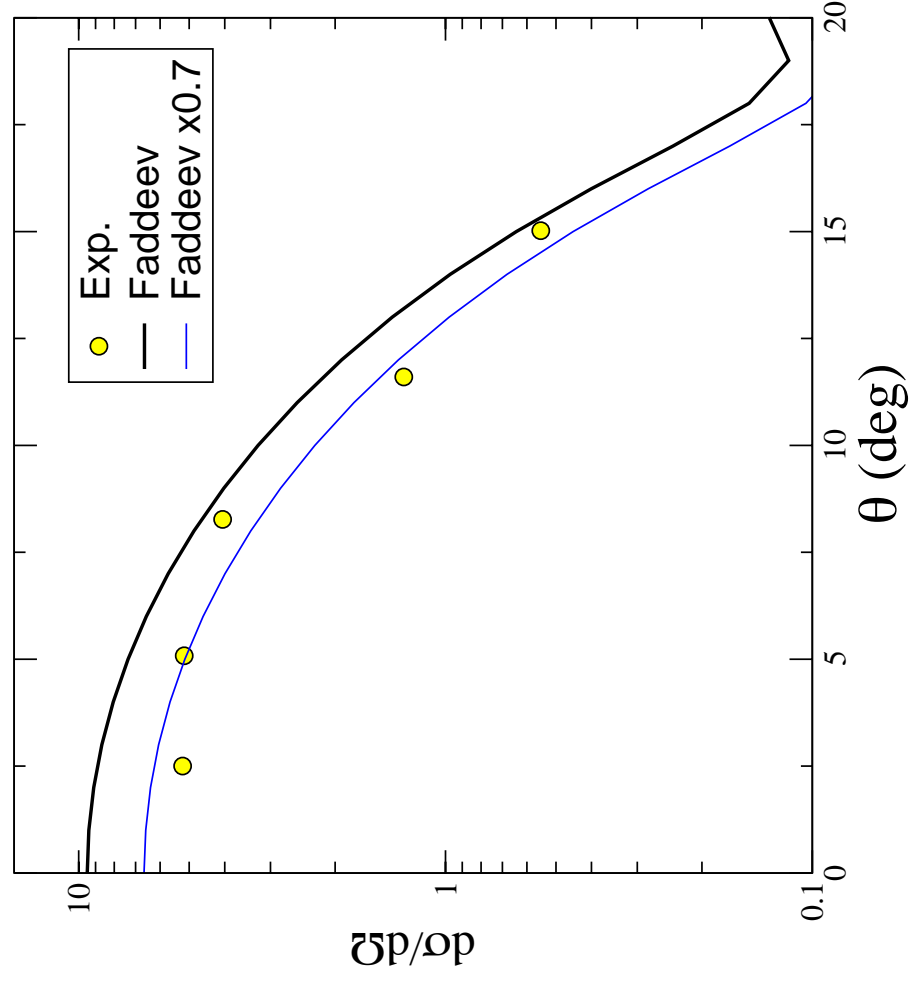
# $p(^{11}\text{Be}, ^{11}\text{Be})p$ @ $E/A \sim 38$ MeV







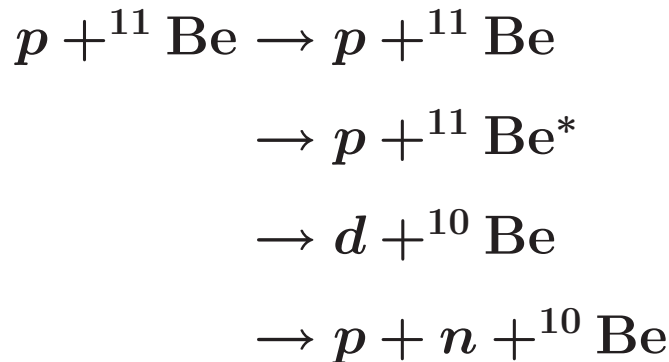
p(11Be,10Be)d @ Ep=35.5 MeV



# CONCLUSIONS

- 1 - No single CDCC calculation matches the exact three-body results.
- 2 - CDCC - BU based on  $n - {}^{10}\text{Be}$  continuum fails in the  $n - p$  quasi-free scattering region of phase space, but gives reasonable results in the region where the cross section is very small but dominated by the low energy states in  ${}^{11}\text{Be}$ .
- 3 - CDCC - TR based on  $n - p$  continuum describes the quasi-free scattering region of breakup but fails in the region of phase space dominated by the  $n - {}^{11}\text{Be}$  low energy states.
- 4 - Exact three-body calculations provide a better description of  $p - {}^{11}\text{Be}$ . Unlike in CDCC,  $p - {}^{11}\text{Be}$  results do not coincide with  $p - {}^{10}\text{Be}$ , and are much closer to  $p - {}^{11}\text{Be}$  data.
- 5 - These are clear signals of the importance of the correct treatment of three-body dynamics.

## How Best Can we Describe



as a three-body problem, much like what is conceptually realized by CDCC calculations.

## MODEL PROBLEM



$p - {}^{10}\text{Be}$  An optical potential with  
spin – orbit interaction + Coulomb

$n - {}^{11}\text{Be}$  A real potential that describes

$2s \frac{1}{2}$  – bound state

$1p \frac{1}{2}$  – excited state

$1d \frac{5}{2}$  – resonance

$n - p$  AV18 or CD Bonn potentials

## Interactions: Spin Dependent

$p - {}^{10}\text{Be}$

### Optical Potential

$$V(r) = -V_o \left[ 1 + e^{\frac{r-R_o}{a_o}} \right]^{-1} - iW_v \left[ 1 + e^{\frac{r-R_v}{a_v}} \right]^{-1} \\ - 4V_{so} \frac{1}{r} \left| \frac{d}{dr} \left[ 1 + e^{\frac{r-R_{so}}{a_{so}}} \right]^{-1} \right| + \text{Coulomb}$$

$$R_i = r_i A^{\frac{1}{3}}$$

Potential	$V_o$	$r_o$	$a_o$	$W_v$	$r_v$	$a_v$	$V_{so}$	$r_{so}$	$a_{so}$
RC1	50.5	1.114	0.57	19.9	1.114	0.50	5.5	1.114	0.57
JC2	18.6	1.53	0.57	11.5	1.0	0.50	7.9	1.0	0.57

$n - p$

### Realistic Interactions

AV18 or CD-Bonn potentials

They describe  $n - p$  scattering up to  $\pi$ -production threshold and deuteron properties.

$n - {}^{10}\text{Be}$ 

## Fitting Low Energy Spectra

$$V(r) = -V_c \left[ 1 + e^{\frac{r-R_c}{a_c}} \right]^{-1} - 4V_{so} \frac{1}{r} \left| \frac{d}{dr} \left[ 1 + e^{\frac{r-R_{so}}{a_{so}}} \right]^{-1} \right|$$

	$L$	$V_c(\text{MeV})$	$r_c(\text{fm})$	$a_c(\text{fm})$	$V_{so}(\text{MeV})$	$r_{so}(\text{fm})$	$a_{so}(\text{fm})$
1A	0	56.413	1.25	0.67	0	1.25	0.67
	odd $\geq 1$	42.498	1.25	0.67	11.953	1.25	0.67
	even $\geq 2$	<b>56.413</b>	1.25	0.67	<b>5.49</b>	1.25	0.67
1B	0	56.413	1.25	0.67	0	1.25	0.67
	odd $\geq 1$	42.498	1.25	0.67	11.953	1.25	0.67
	even $\geq 2$	<b>49.77</b>	1.25	0.67	<b>11.953</b>	1.25	0.67
2A	0	62.52	1.20	0.60	0	1.20	0.60
	odd $\geq 1$	45.35	1.20	0.60	10.571	1.20	0.60
	even $\geq 2$	<b>62.52</b>	1.20	0.60	<b>5.25</b>	1.20	0.60
2B	0	62.52	1.20	0.60	0	1.20	0.60
	odd $\geq 1$	45.35	1.20	0.60	10.571	1.20	0.60
	even $\geq 2$	<b>56.41</b>	1.20	0.60	<b>10.571</b>	1.20	0.60

## Spectra of $^{11}\text{Be}$

State	1A(MeV)	1B(MeV)	2A(MeV)	2B(MeV)
$1d \frac{5}{2}$	$1.274 - i 0.201/2$	$1.275 - i 0.206/2$	$1.290 - i 0.169/2$	$1.274 - i 0.167/2$
$1p \frac{1}{2}$	$- 0.1831$	$- 0.1831$	$- 0.1831$	$- 0.1831$
$2s \frac{1}{2}$	$- 0.5029$	$- 0.5029$	$- 0.5064$	$- 0.5064$
$1p \frac{3}{2}$	$- 6.812^*$	$- 6.812^*$	$- 6.812^*$	$- 6.812^*$
$1s \frac{1}{2}$	$- 28.73^*$	$- 28.73^*$	$- 32.74^*$	$- 32.74^*$

\* states to be removed due to Pauli exclusion

$$^{11}\text{Be}(\text{rms}) \simeq 2.98 \text{ fm} \quad (\text{Exp} = 2.90)$$

$$0.250 < B(E_1) < 0.273 e^2 \text{ fm}^2 \quad (\text{Exp} = 0.116)$$

Neutron Separation Energy in  $^{10}\text{Be} = 6.812 \text{ MeV}$

## Partial Waves in the Calculation

$$\boxed{p - {}^{10}\text{Be}}$$

$$\ell \leq 11$$

$$|\ell - \frac{1}{2}| \leq j \leq \ell + \frac{1}{2}$$

$$\boxed{n - p}$$

$$I \leq 3 + {}^3\text{F}_4$$

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$${}^1\text{S}_0 ; {}^3\text{P}_0 ; {}^3\text{S}_1 - {}^3\text{D}_1 ; {}^1\text{P}_1, {}^3\text{P}_1 ; {}^1\text{D}_2, {}^3\text{D}_2 ; {}^3\text{P}_2 - {}^3\text{F}_2$$

$$0^+ ; 0^- ; \quad 1^+ \quad ; \quad 1^- \quad ; \quad 2^+ \quad ; \quad 2^-$$


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$${}^3\text{D}_3 - {}^3\text{G}_3 ; {}^1\text{F}_3, {}^3\text{F}_3 ; {}^1\text{G}_4, {}^3\text{G}_4 ; {}^3\text{F}_4 - {}^3\text{H}_4$$

$$3^+ \quad ; \quad 3^- \quad ; \quad 4^+ \quad ; \quad 4^-$$


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$${}^3\text{G}_5 - {}^3\text{I}_5 \quad ; \quad {}^1\text{H}_5, {}^3\text{H}_5$$

$$5^+ \quad ; \quad 5^-$$


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$$\boxed{n - {}^{10}\text{Be}}$$

$$\ell \leq 4$$

$$|\ell - \frac{1}{2}| \leq j \leq \ell + \frac{1}{2}$$


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Total Three-Body  $J \leq 15$  (elastic & transfer)

$J \leq 30$  (breakup)

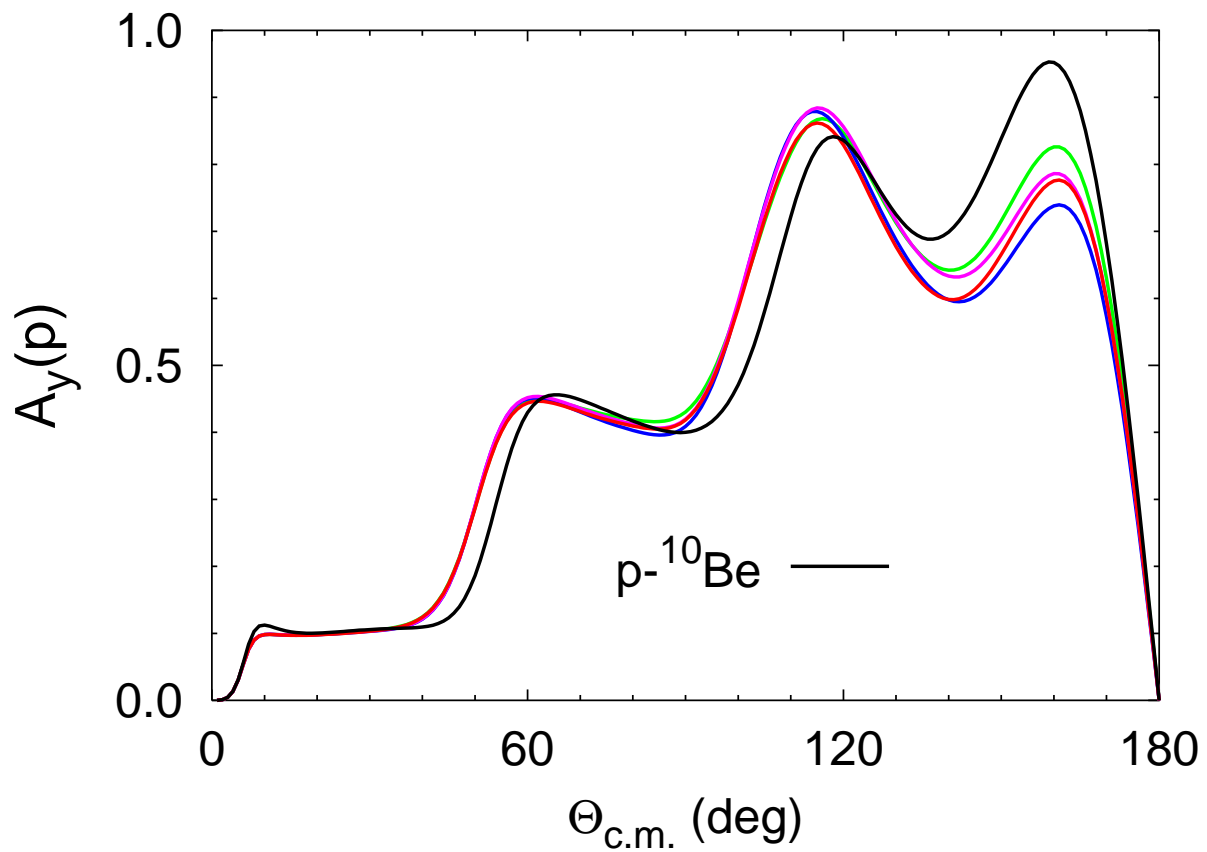


# Elastic Scattering

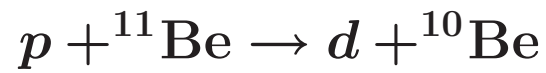
$p - {}^{10}\text{Be} = \text{RC1}$

$n - {}^{10}\text{Be} = 1\text{A}(\text{red}); 1\text{B}(\text{green}); 2\text{A}(\text{blue}); 2\text{B}(\text{magenta})$

$n - p = \text{CD Bonn}$



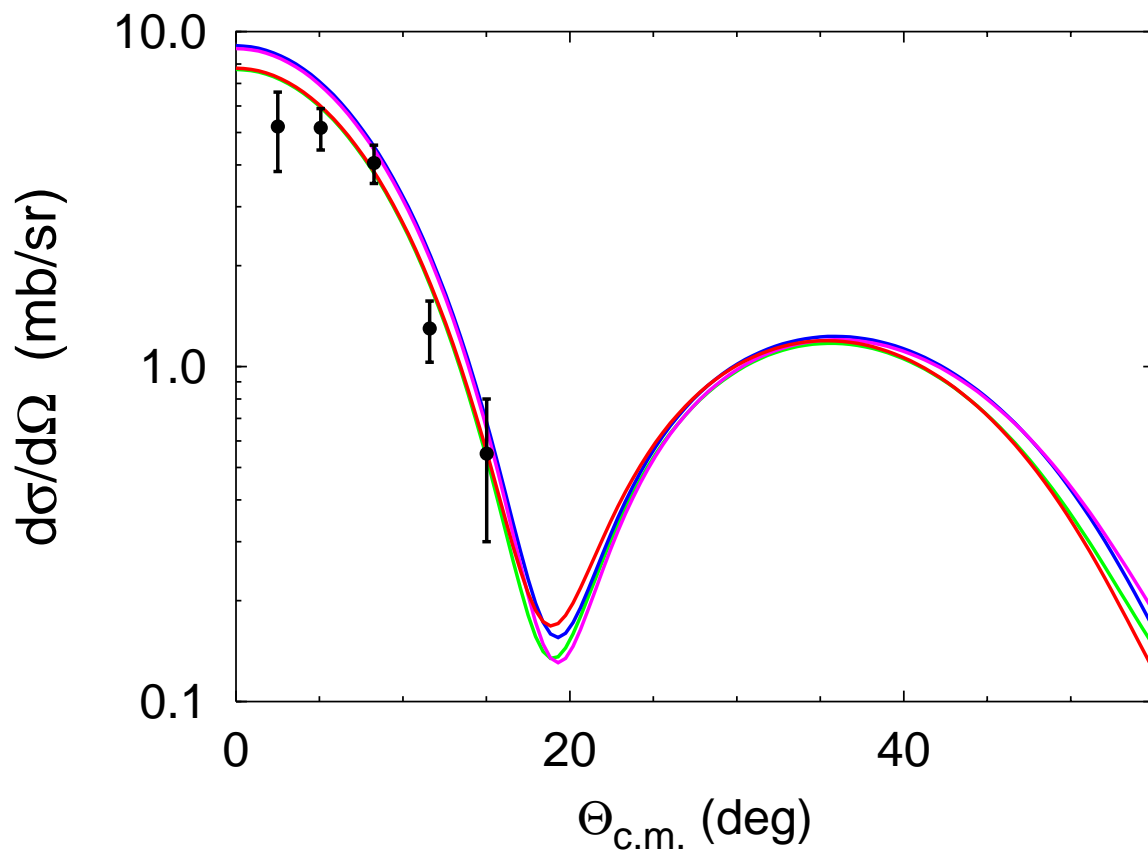
# Transfer Reaction



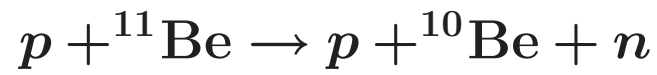
$p - {}^{10}\text{Be} = \text{RC1}$

$n - {}^{10}\text{Be} = 1\text{A}(\text{red}); 1\text{B}(\text{green}); 2\text{A}(\text{blue}); 2\text{B}(\text{magenta})$

$n - p = \text{CD Bonn}$



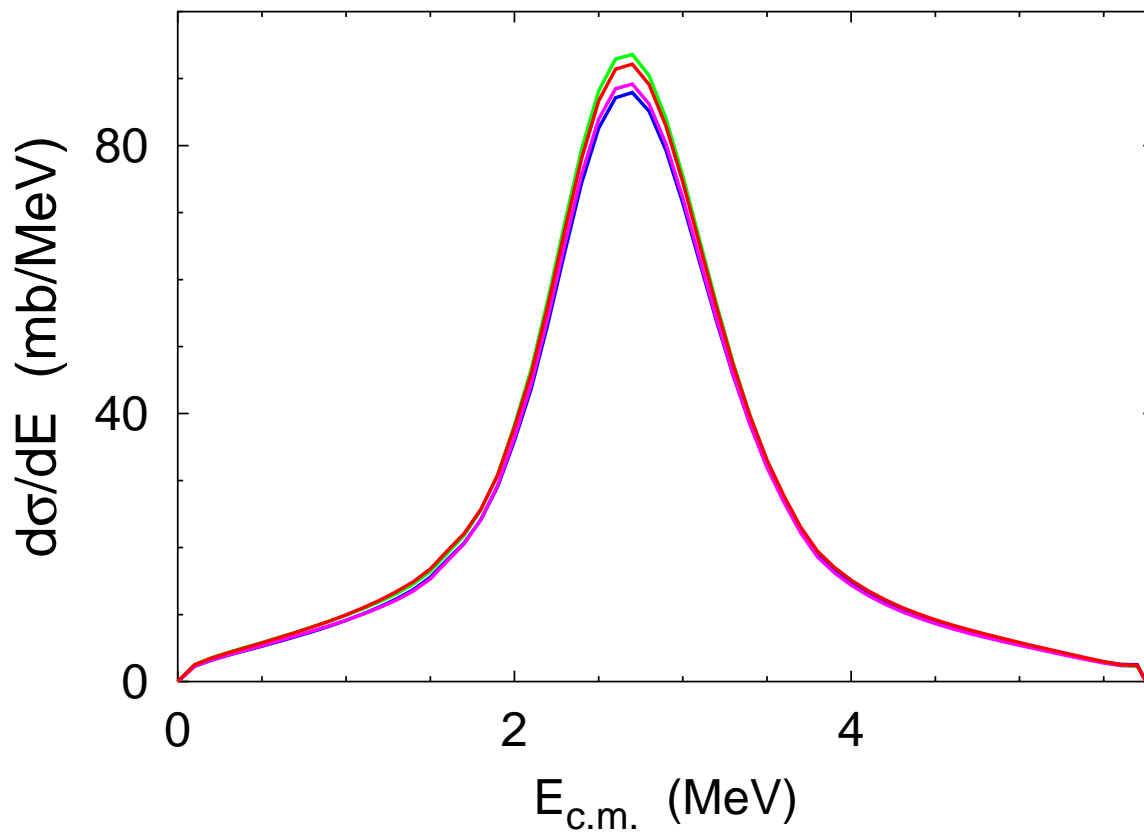
# Breakup



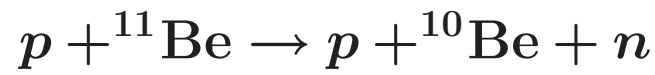
$p - {}^{10}\text{Be} = \text{RC1}$

$n - {}^{10}\text{Be} = 1\text{A}(\text{red}); 1\text{B}(\text{green}); 2\text{A}(\text{blue}); 2\text{B}(\text{magenta})$

$n - p = \text{CD Bonn}$



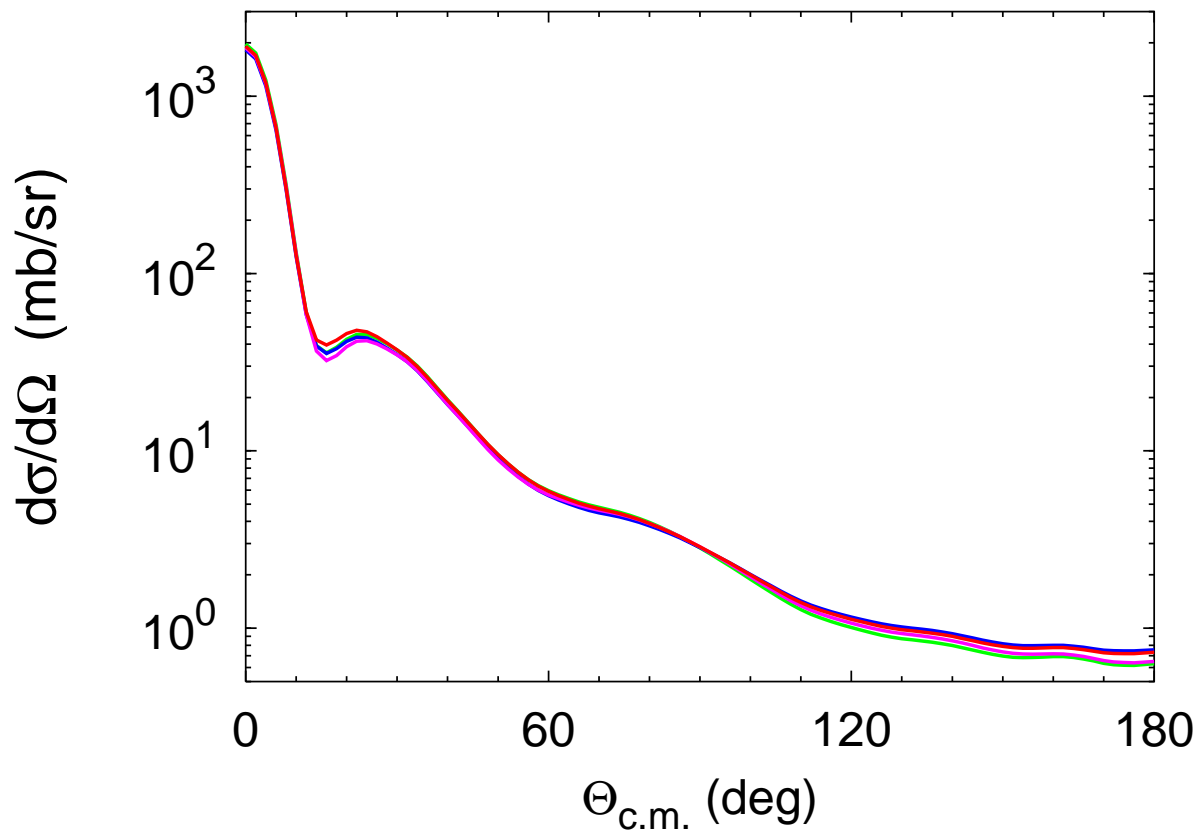
# Breakup



$p - {}^{10}\text{Be} = \text{RC1}$

$n - {}^{10}\text{Be} = 1\text{A}(\text{red}); 1\text{B}(\text{green}); 2\text{A}(\text{blue}); 2\text{B}(\text{magenta})$

$n - p = \text{CD Bonn}$

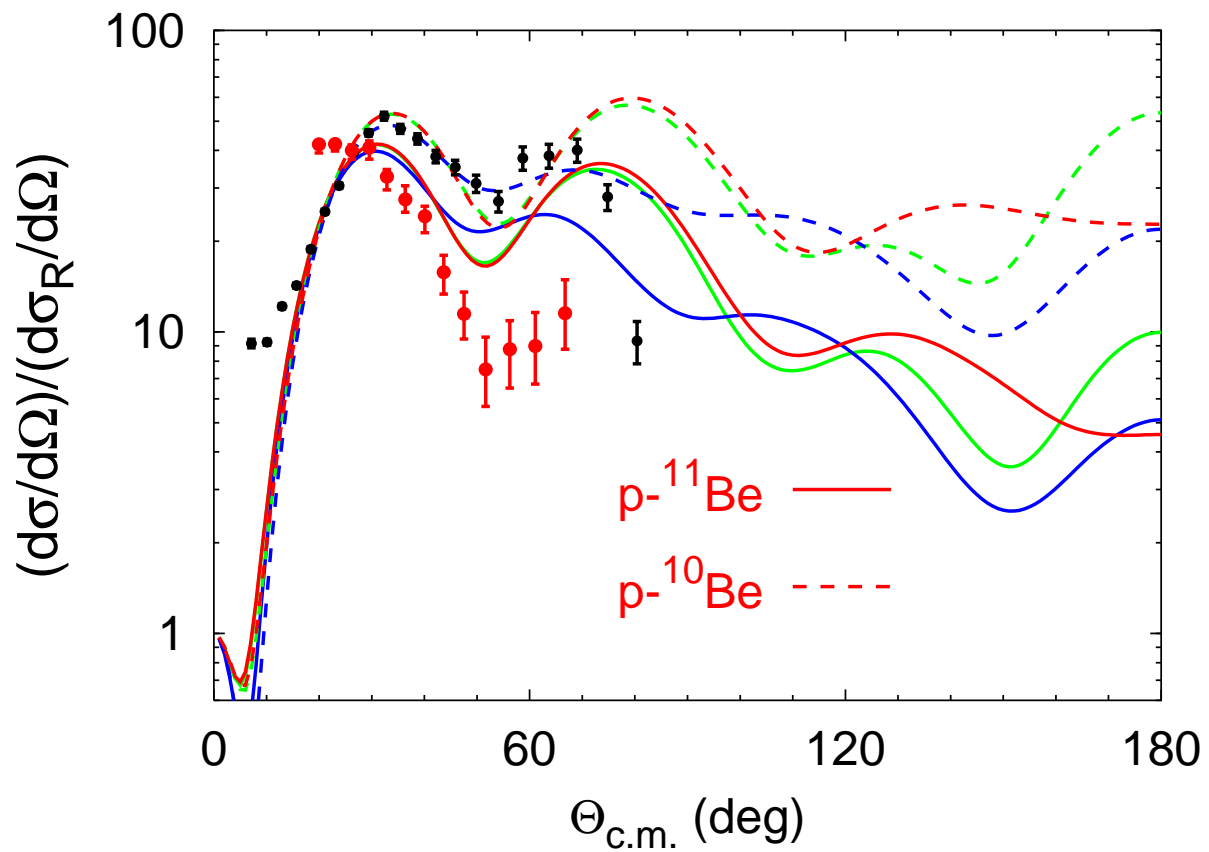


# Elastic Scattering

$p - {}^{10}\text{Be} = \text{RC1}(\text{red}); \text{JC2}(\text{blue}); \text{RC1} - V_{so} = 0(\text{green})$

$n - {}^{10}\text{Be} = 1\text{A}$

$n - p = \text{CD Bonn}$

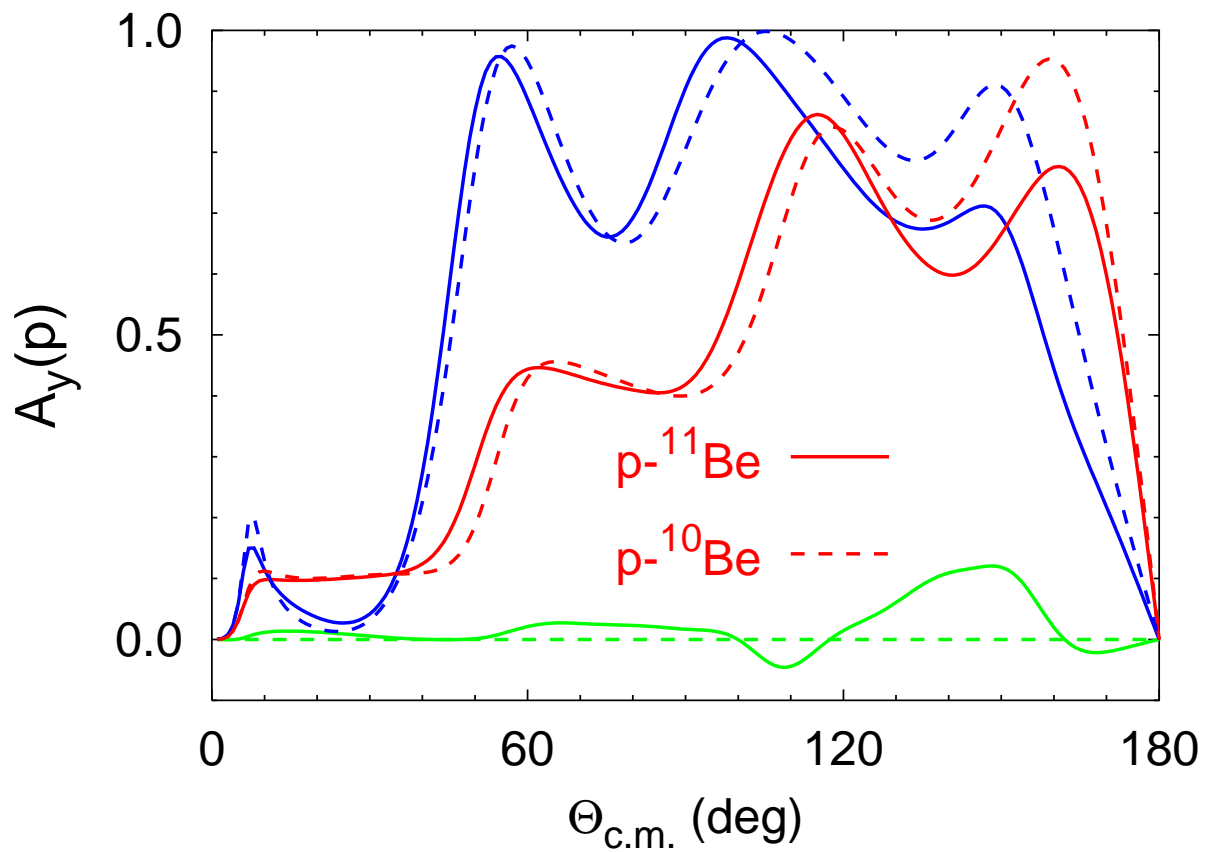


# Elastic Scattering

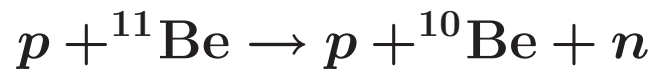
$p - {}^{10}\text{Be}$  = RC1(red); JC2(blue); RC1 -  $V_{so} = 0$ (green)

$n - {}^{10}\text{Be}$  = 1A

$n - p$  = CD Bonn



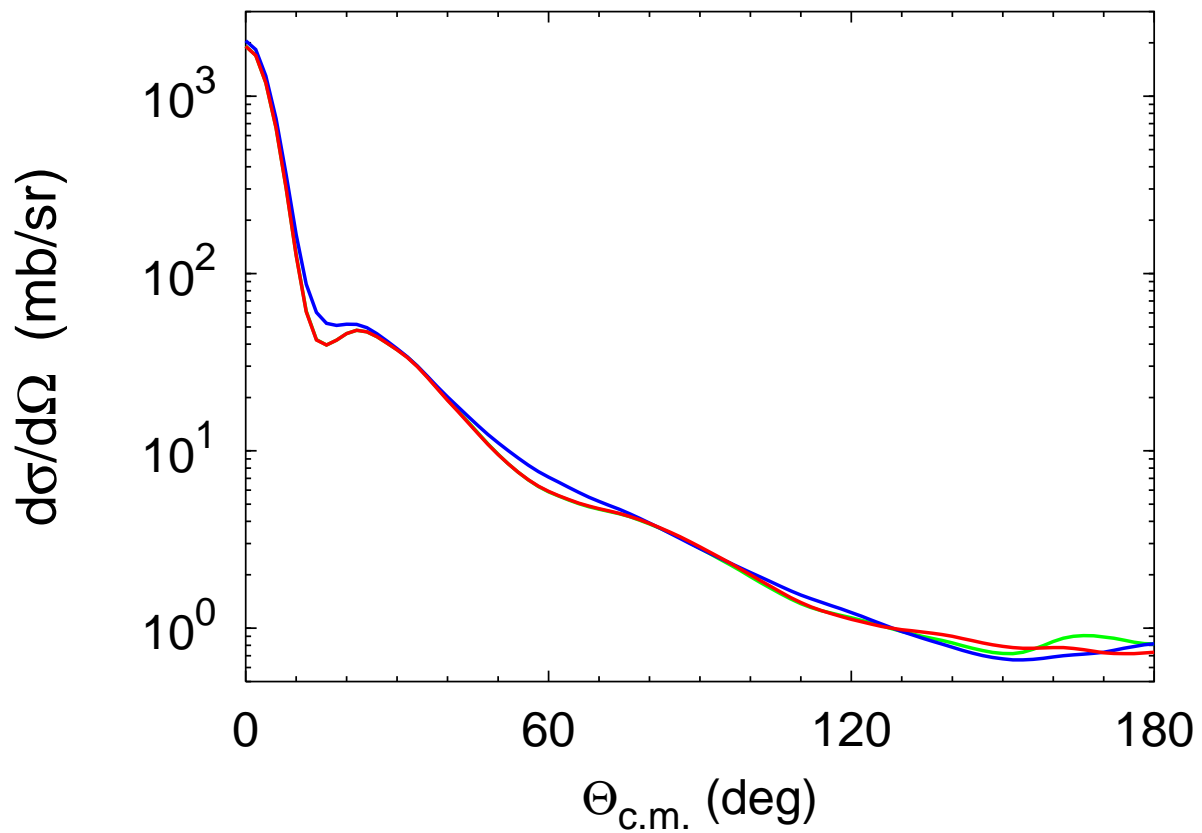
# Breakup



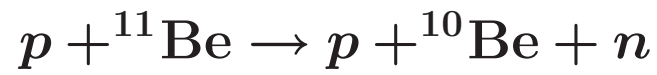
$p - {}^{10}\text{Be} = \text{RC1}(\text{red}); \text{JC2}(\text{blue}); \text{RC1} - V_{so} = 0(\text{green})$

$n - {}^{10}\text{Be} = 1\text{A}$

$n - p = \text{CD Bonn}$



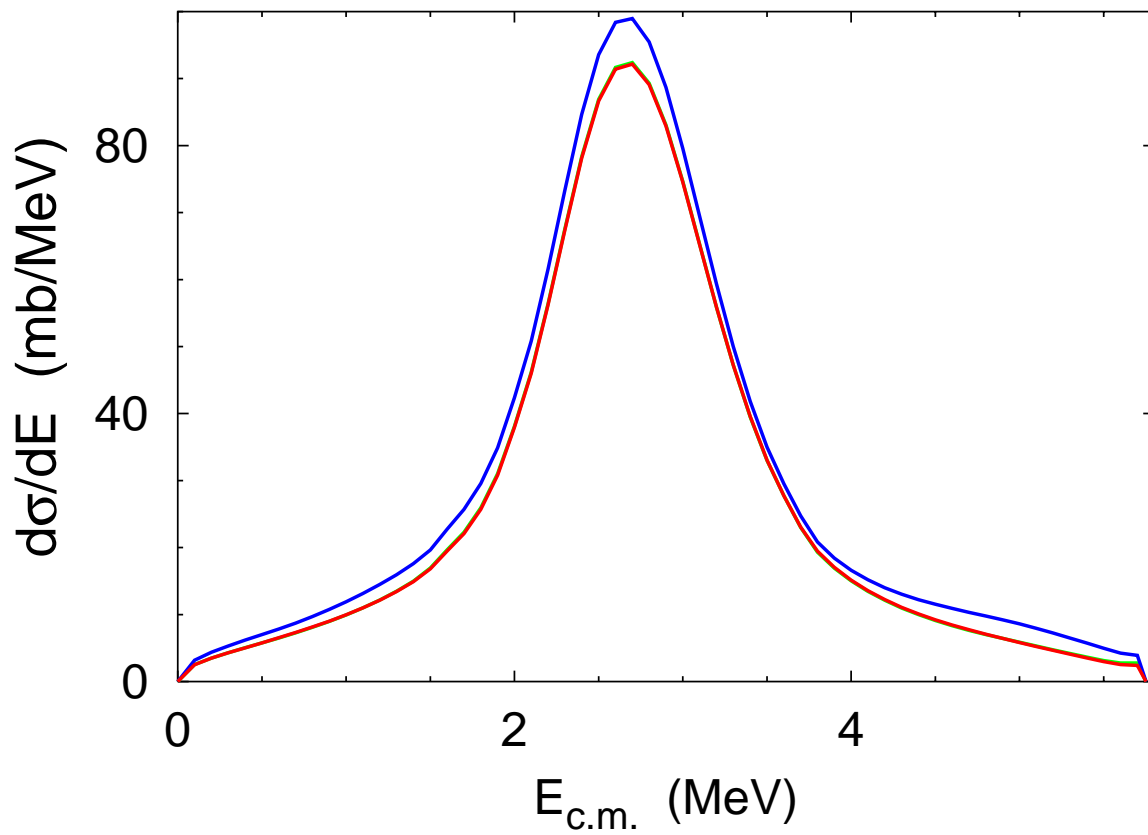
# Breakup



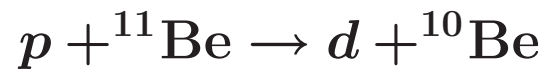
$p - {}^{10}\text{Be} = \text{RC1}(\text{red}); \text{JC2}(\text{blue}); \text{RC1} - V_{so} = 0(\text{green})$

$n - {}^{10}\text{Be} = 1\text{A}$

$n - p = \text{CD Bonn}$



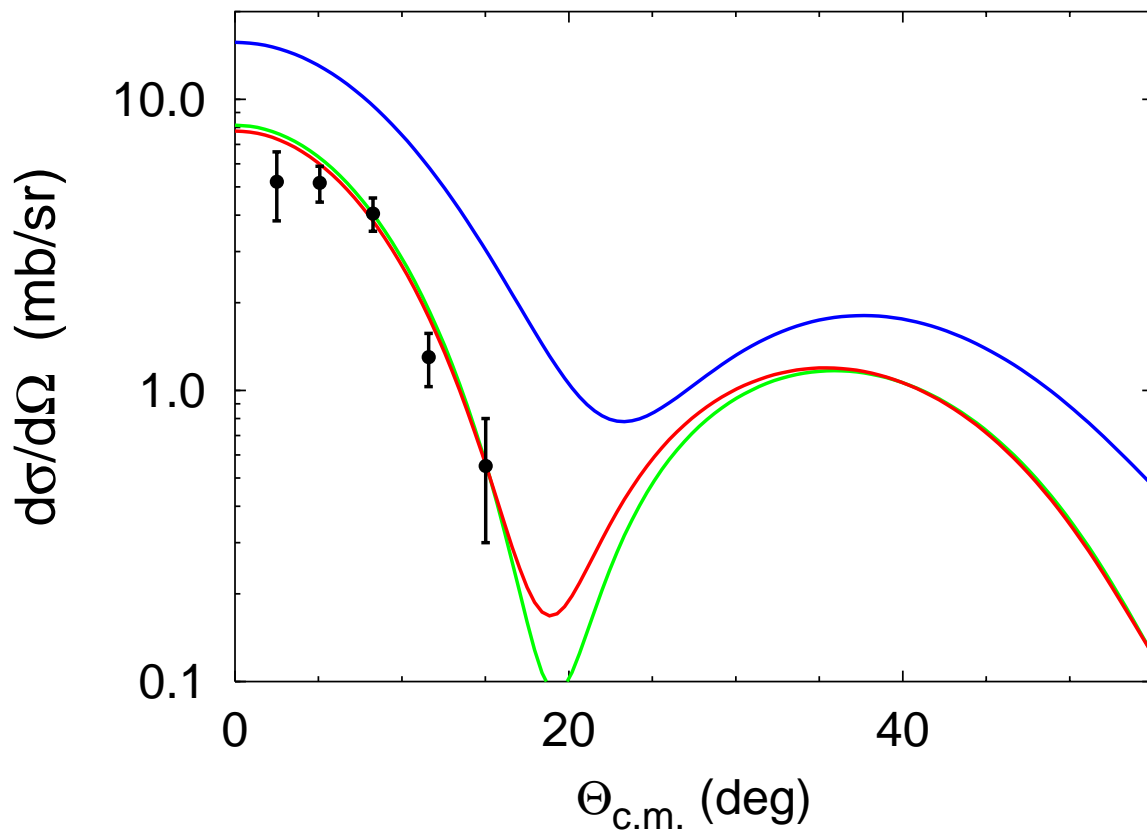
# Transfer Reaction



$p - {}^{10}\text{Be} = \text{RC1}(\text{red}); \text{JC2}(\text{blue}); \text{RC1} - V_{so} = 0(\text{green})$

$n - {}^{10}\text{Be} = 1\text{A}$

$n - p = \text{CD Bonn}$



## CONCLUSIONS

- 1 - We can successfully apply exact three-body calculations to direct nuclear reactions.
- 2 - Unlike CDCC we can calculate elastic, transfer and breakup reactions simultaneously.
- 3 - Unlike present CDCC calculations  $p - {}^{11}\text{Be}$  results are clearly separated from  $p - {}^{10}\text{Be}$  and closer to the corresponding data.
- 4 - The data for the transfer reaction is well described by the calculation, even in this single particle state approach to the ground state of  ${}^{11}\text{Be}$ . (No apparent need for a spectroscopic factor in some of the models).
- 5 - Results depend strongly on the  $p - {}^{11}\text{Be}$  optical potential which happens to be the least constrained interaction by existing  $p - {}^{10}\text{Be}$  data.
- 6 - Results are not very sensitive to the  $n - {}^{10}\text{Be}$  or  $n - p$  interactions.
- 7 - The transfer reaction clearly rejects the choice of JC2 for the  $p - {}^{10}\text{Be}$  optical potential parametrization.